Statistical Process Control for Charting Multiple Sources of Variation with an Application to Neutron Tube Production

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Prepared by
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Albuquerque, New Mexico 87185 and Livermore, California 94550
for the United States Department of Energy
under Contract DE-AC04-76DP00789
Multiple sources of variation will often affect the stability of a manufacturing process. Items from different batches may vary because of variation both within a batch and among different batches. Potential sources of variation include within run, run-to-run and week-to-week differences in a manufacturing process. If multiple sources of variation are present, traditional control chart methods may not be appropriate. In this report we develop control charts for monitoring these sources of variation as well as the process average. An example of how to use the control charts is given, using Field 89 data from functional testing of the MC3854 neutron tube.
1. Introduction

Frequently in a manufacturing setting, multiple sources of variation will affect the stability of the process. Units within a batch will vary as will units from different batches. Sources of variation in a manufacturing process include differences in production runs or setups and differences over time. If such multiple sources of variation are present, traditional control chart methods may not be appropriate. Control chart methods that explicitly recognize and monitor the multiple sources of variation are needed to evaluate process performance. The purpose of this report is to present a statistical model that takes into account multiple sources of variation, and to develop control charts for monitoring these sources of variation as well as the process average.

In what follows, these methods will be developed and illustrated with data from the MC3854 neutron tube. The next section will briefly discuss some of the background theory for control charts. Following that, a mathematical model that can be used to characterize data with multiple sources of variation is given and applied to the MC3854 neutron tube data. Also, the natural control chart extensions that arise from the model are discussed. The analysis of one year's data for the MC3854 tube as related to the mathematical model is then summarized. Conclusions about the use of statistical process control (SPC) and additional applications of control charts for monitoring multiple sources of variation are addressed in the final section.

2. Control Chart Background

The intent in this section is not to go into great detail concerning the theory and use of control charts in general, but rather to review aspects that are important with respect to a process with multiple sources of variation like the fabrication of neutron tubes. There are many books and articles that cover the basics of control charts. The Western Electric Handbook (1956), Wheeler and Chambers (1986), and Montgomery (1991) are recommended for both the basics of control charting as well as the philosophy behind control charts. Woodall and Thomas (1992) present methods for statistical process control with several components of variation, although the control charts they develop are somewhat different than those presented here.

Shewhart (1931) characterized process variation as being either controlled variation or uncontrolled variation. Controlled variation is characterized by a consistent and stable pattern over time. Such
variation is attributed to "chance" causes, and can only be affected by fundamental changes in the production system. Uncontrolled variation is characterized by a changing or inconsistent pattern of variation. These changes in the pattern of variation are attributed to "assignable" causes. These are special factors that change from time to time and make the process inconsistent and unstable. The identification and removal of these factors will improve the process. When the process operates as intended, only controlled variation will be present.

Shewhart developed control charts as a tool to characterize a process with respect to these two sources of variation. A process "in control" exhibits only controlled variation, while a process "out of control" exhibits both sources of variation. "In control" or "out of control" conditions are established by considering consistency and stability, which are characterized statistically using groups of measurements. The measurements come from units that Shewhart referred to as "rational subgroups," which are subgroups intended to contain only controlled variation. Thus, in control chart theory an essential ingredient in quantifying variability is establishing rational subgroups. In standard control chart use, the variation within the rational subgroups becomes the standard against which the variation from group-to-group is judged. A process "in control" is thus one in which not only the variation within subgroups remains consistent (statistically predictable) from group to group, but also one in which the variation in measurement averages from one group to the next is consistent with the variation within groups.

For neutron tubes a readily available and natural subgroup to consider is that consisting of the units from a single exhaust run. The exhaust run is one of the key processing steps in the manufacturing of neutron tubes. A number of tubes (10 or 11 depending on the exhaust system) are exhausted at the same time in one of four exhaust systems. A Field 89 measurement is taken later on each tube to determine the acceptability of that tube. To use the group of units in one run as the "rational" subgroup in a control chart scheme, however, implicitly categorizes any additional sources of variation (such as run-to-run or week-to-week variation) as "uncontrolled" variation, whose existence would be attributed to assignable causes rather than chance causes. Thus, with a traditional control chart application there is a greater risk of "out of control" signals if there are in fact some additional components of natural variation, even if this additional variation is stable and consistent.

Rather than have control charts that show "out of control" conditions because of natural and acceptable run-to-run and/or week-to-week variation, these sources are recognized and are
incorporated into the control chart monitoring schemes in the next section. The introduction of additional charts is necessary to monitor the additional sources of variation. The theory behind and the rationale for additional charts for the neutron tube data is given in the following section.

3. Model for MC3854 Data

For the neutron tube manufacturing process, we will assume that week-to-week variation and exhaust run-to-exhaust run variation are inherent to the process. Under this assumption, a model can be constructed to explicitly account for these sources of variation, and control charts that monitor the multiple sources of variation can be developed. These charts can be used to detect unusual weeks of production or unusual exhaust runs within a week and thus provide clues about assignable causes that may lead to process improvement.

The assumed model under which the control charts discussed here are derived is given by

\[ y_{ijk} = \mu + w_i + r_j + \epsilon_{ijk}, \]

where \( y_{ijk} \) corresponds to a measurement taken in week \( i = 1, 2, \ldots, n \), on exhaust run \( j = 1, 2, \ldots, m_j \) within week \( i \), and on unit \( k = 1, 2, \ldots, p_{ij} \) within run \( j \). The subscripts on the limits for indices \( j \) and \( k \) signify that there could be a different number of runs within a week as well as different number of units tested within a run. In the above model, \( \mu \) represents the overall average of the Field 89 data, \( w_i \) represents the random effect associated with the \( i \)th week, \( r_j \) represents the random effect associated with the \( j \)th run within the \( i \)th week, and \( \epsilon_{ijk} \) is the random error term associated with the \( k \)th tube within the \( j \)th run of the \( i \)th week.

In the discussion that follows it is assumed that \( p_{ij} \) is constant for all the runs in every week (\( p_{ij} = p \)). It is further assumed that the \( w_i \)'s are random quantities from a normal distribution with mean 0 and variance \( \sigma_w^2 \), the \( r_j \)'s are random quantities from a normal distribution with mean 0 and variance \( \sigma_r^2 \), and the \( \epsilon_{ijk} \)'s are normally distributed with mean 0 and variance \( \sigma^2 \). Independence of the \( w_i \)'s, \( r_j \)'s and \( \epsilon_{ijk} \)'s is assumed throughout.

Typically, the parameters \( \mu \), \( \sigma_w \), \( \sigma_r \), and \( \sigma \) will not be known and must be estimated from the data. The overall mean \( \mu \) is estimated by \( \overline{y} \), the grand sample mean. A variance components routine in a statistical analysis package, such as SAS, should be used to compute estimates of \( \sigma_w \),
\( \sigma_r \), and \( \sigma \), denoted by \( \hat{\sigma}_w \), \( \hat{\sigma}_r \), and \( \hat{\sigma} \), respectively. It is important to first check the data for obvious outliers because they can have a substantial effect on the estimates of the variance components.

We recommend monitoring the unit-to-unit variation within a run, the run-to-run variation within a week, and the weekly averages as follows:

1. The sample standard deviation within a run, given by

\[
s_{ij} = \left( \frac{1}{(p-1)} \sum_{k=1}^{p} (y_{ijk} - \bar{y}_{ij})^2 \right)^{1/2},
\]

should be compared to a centerline of \( c_4 \sigma \), with lower and upper limits given by \( B_5 \sigma \) and \( B_6 \sigma \) respectively. Here \( \bar{y}_{ij} \) is the average of the jth run within the ith week. The constants \( c_4 \), \( B_5 \), and \( B_6 \) are control limit constants which depend on the number of measurements, \( p \), that make up the subgroup. Tables of their values are given in most control chart reference books and for subgroups of size 10 or less are reproduced here in Table 1.

2. The sample standard deviations calculated from run averages within a week, given by

\[
s_i = \left( \frac{1}{(m_1-1)} \sum_{j=1}^{m_1} (\bar{y}_{ij} - \bar{y}_{i.})^2 \right)^{1/2},
\]

should be compared to a center line of \( c_4 \sigma_{ra} \), a lower limit of \( B_5 \sigma_{ra} \), and an upper limit of \( B_6 \sigma_{ra} \). Here the constants \( c_4 \), \( B_5 \), and \( B_6 \) depend on \( m_1 \), the number of runs for the ith week. Tables of their values are also reproduced here in Table 1, for \( m_1 \leq 10 \). Here \( \bar{y}_{ij} \) is the average of the jth run within the ith week and \( \bar{y}_{i.} \) is the average of the ith week. The standard deviation of the run averages \( \bar{y}_{ij} \) is given by

\[
\sigma_{ra} = \left( \sigma_r^2 + \sigma^2 / p \right)^{1/2}.
\]

3. The weekly averages, denoted \( \bar{y}_{i.} \), should be plotted and compared to a mean of \( \mu \) and control
limits of $\mu \pm 3\sigma_{wa}$, where $\sigma_{wa}$, the standard deviation of weekly averages, is given by

$$\sigma_{wa} = \left(\frac{\sigma_w^2 + \sigma_r^2}{m_i} + \frac{\sigma^2}{pm_i}\right)^{1/2}.$$

Note that $\sigma_{wa}$ depends upon $m_i$, the number of runs in week $i$.

Given estimates of $\mu$, $\sigma_w$, $\sigma_r$, and $\sigma$, the three charts (for within run standard deviations, run-to-run standard deviations, and weekly averages) together can be used to monitor the process. In summary, the three charts should be constructed as follows:

1. Chart of standard deviations within runs: Plot the $s_{ij}$'s on a control chart with a centerline of $c_4\sigma$, a lower limit of $B_3\sigma$, and an upper limit of $B_6\sigma$.

2. Chart of standard deviations calculated from run averages within a week: Plot the $s_1$'s on a control chart with a centerline of $c_4\sigma_{ra}$, a lower limit of $B_3\sigma_{ra}$, and an upper limit of $B_6\sigma_{ra}$.

3. Chart of weekly averages: Compute the $\bar{y}_i$'s and plot them on a control chart with centerline at $\bar{y}$ and control limits at

$$\bar{y}_{\ldots} \pm 3\sigma_{wa}.$$

Chart number one above should be examined first to see if any of the $s_{ij}$'s fall outside the control limits. An out of control point on this chart would indicate an unusual amount of variation (higher or lower than usual) within one run. Any $s_{ij}$ value that falls outside the control limits should be investigated to see if an "assignable" cause can be identified. If an assignable cause can be identified and removed from the process, the data that caused the out of control point should be deleted from the analysis, and the control limits for all three charts should be recalculated. An $s_{ij}$ value falling below the lower limit would indicate an unusually low amount of variation. In this case, identifying an "assignable cause" of lower process variation could lead to a sustainable process improvement. If none of the $s_{ij}$'s fall outside the control limits, chart number two above should next be examined to see if any of the $s_1$'s fall outside their control limits. An out of control point on this chart would indicate an unusual amount of variation within one week. Any $s_1$ value that falls outside the control limits should also be investigated to see if an assignable cause can be
identified. If an assignable cause can be identified and removed from the process, the data that caused the out of control point should be deleted from the analysis, and the control limits for charts one and two only should be recalculated. If none of the $\delta_i$'s fall outside the control limits, chart number three above should next be examined to see if any of the $\bar{y}_{i}$. fall outside their control limits. An out of control point on this chart would indicate an unusual value for one weekly average. Any $\bar{y}_{i}$. value that falls outside the control limits should then be investigated to see if an assignable cause can be identified and removed from the process.

The above discussion outlines a procedure for using control charts to determine if the process was “in control” over the period of time during which the data were collected. Once the historical data have been analyzed and appropriate control limits have been established, these control limits can be used for the purpose of ongoing control of the process. If the process parameters $\mu$, $\sigma_w$, $\sigma_r$, or $\sigma$ change significantly, then after any assignable causes are removed from the process, the control limits should be recalculated according to the above procedure.

Construction and interpretation of these charts will be illustrated with MC3854 Field 89 data.

4. Summary of MC3854 Data

In order to illustrate the above charting procedures, Field 89 data from functional acceptance testing of MC3854 taken in 1987 were analyzed. The data consist of the Field 89 measurements from this year. This section presents results from Field 89 measurements for units from exhaust system 4. For this exhaust system the number of units per run was 10 ($p=10$), and the number of runs per week varied. Before the analysis was done, all high voltage breakdowns were removed from the data. Figures 1-3 illustrate the control charts as developed in the previous section. Figure 1 is the plot of within run standard deviations, Figure 2 is the plot of run-to-run standard deviations, and Figure 3 is a plot of both individual Field 89 measurements and weekly averages. Before discussing these charts it is necessary to give some additional background.

The data were analyzed assuming the model $y_{ijk} = \mu + \beta \cdot \text{xdim} + w_i + r_j + \epsilon_{ijk}$. This model differs from that discussed earlier in that an explanatory variable, xdim, is included. The variable xdim is a dimension in the ion source. Linear regression analysis with the variable xdim showed that it alone is capable of explaining about one third of the total variation in the Field 89 measurements. For this reason, the variation due to xdim was removed before the charts were constructed.
Estimates of the \( x_{\text{dim}} \) dependency were obtained for exhaust system 4, resulting in \( \hat{b} = .224 \). The values for \( x_{\text{dim}} \) ranged between 290 and 330. Each of the data points \( y_{ijk} \) were transformed to \( y_{ijk} - .224 \cdot (x_{\text{dim}ijk} - 310) \), which adjusts the measurements to \( x_{\text{dim}} = 310 \), and it is this quantity that is analyzed according to the variance component model given in the earlier section.

These data were analyzed using variance component routines in the statistical data analysis package SAS. The resulting estimates for exhaust system 4 Field 89 measurements are \( \hat{\mu} = 42.7 \), \( \hat{\sigma}_w = .67 \), \( \hat{\sigma}_r = 1.01 \), and \( \hat{\sigma} = 2.18 \). These values result in \( \hat{\sigma}_{ra} = 1.22 \).

Figure 1 shows the control chart for the within run standard deviations. For subgroups of size 10, \( c_4 = .973 \), \( B_5 = .276 \), and \( B_6 = 1.669 \). Therefore the centerline is \( .973 \cdot 2.18 = 2.12 \), the lower limit is \( .276 \cdot 2.18 = .60 \), and the upper limit is \( 1.669 \cdot 2.18 = 3.64 \). In Figure 1 each within run standard deviation is plotted versus the exhaust week. If control chart procedures were being done in real time, each run standard deviation would be plotted separately as the next point on the chart.

Figure 2 shows the control chart for the run-to-run standard deviations. In this case the number of runs per week ranged from 1 to 7. For weeks with a single run, no standard deviation could be calculated and therefore on this chart no points would be plotted for those weeks. The standard deviation being estimated, \( \sigma_{ra} \), is assumed known and equal to \( \hat{\sigma}_{ra} \). The coefficient \( c_4 \) for subgroups of size 2 to 7 ranges from .798 to .959 and this accounts for the centerline variation between .97 and 1.17. Similarly, the coefficient \( B_6 \) ranges between 1.806 and 2.606 and thus when multiplied by 1.22 results in variation between 2.20 and 3.18.

In Figure 3 the connected dots are the weekly averages. The asterisks are the individual run averages. The control limits are for the weekly averages. Control limits on individual runs would correspond to the plateau in weeks 30 to 37, corresponding to \( m_1 = 1 \).

These control charts indicate that for this set of yearly data, there was not a lot of uncontrolled variation. In weeks 20 and 40 there were runs that displayed uncharacteristic within run variation (see Figure 1). This could be because of a single unit out of the 10 units in a run, or it could be an overall increase in variation within the runs. If an assignable cause of the two within-run standard deviations falling outside the control limits could be identified and eliminated, then those two data points would be deleted and the limits for each of the three charts would be recalculated. In week 46, the run-to-run standard deviation was outside the control limit (see Figure 2). If an assignable
cause for this occurrence could be identified and eliminated, then that data point would be deleted and the limits for charts two and three would be recalculated. Also, in week 46 one run was unusually high (see Figure 3). Because the within run standard deviations for week 46 were as expected, this means that all 10 units in that run had readings that were higher than would be expected if the process was operating within its capability.

Besides providing input to the construction of control limits, the estimates $\hat{\sigma}_w$, $\hat{\sigma}_r$, and $\hat{\sigma}$ indicate where emphasis should be placed in terms of reducing total process variability. For the Field 89 data, $\hat{\sigma}$ is roughly twice the size of $\hat{\sigma}_r$ and roughly three times the size of $\hat{\sigma}_w$. So efforts to reduce variability should be concentrated on the within run variation since it accounts for the majority of the total variation, after the effect of $\text{xdim}$ is removed from the process.

5. Interpretation of MC3854 Results

Consider the lowest run in week 17 and the highest run in week 46 from Figure 3. From Figure 1 it can be determined that the variation among the 10 units in a run, for both cases, is well within the exhaust system's capability. The run average for the week 17 low point is also within expected behavior of the process (individual run averages are compared to the plateaus in weeks 30-37, which correspond to $m_i=1$). The run average for the week 46 high point is well beyond expected behavior. If one of the units in week 17 happened to fall below a minimum requirement and one (or more) of the units in week 46 was above a maximum requirement, our reaction would not be the same. In the first case the process is operating as expected. Thus, if the process is unacceptable, there will have to be some fundamental changes to get an improvement. But in the second case, the process is not operating as expected, and hence it is likely that an assignable cause could be identified and steps put into place that would minimize its chance of recurrence.

The arguments of the previous paragraph are tied to the idea of process improvements. It is the case that the information provided by the control charts is the most useful in the production environment. However, there are also implications about the acceptability of the product. For example, even if only a few of the 10 units in the high run in week 46 failed some specification, all their sister units would be suspect because of the clear indication that some assignable cause of variation for the whole run exists. This would not be the case for the low run in week 17.
6. Conclusions and Additional Applications

In modeling the MC3854 measurements several sources of variation were explicitly recognized. It should be noted that some of those sources may not be significant. This was the case with the sample data. The estimated week-to-week variation ($\sigma^2_{ww}$) was not statistically significant in the data analyzed. Thus, the variation seen from one week to the next would not be greatly different even if the week-to-week variation were not modeled. The full model was retained in the example to illustrate the complete approach.

By assuming different sources of variation in a model one should question what that means with respect to the process being modeled. For example, if week-to-week variation exceeds the amount one would expect, then the aspects of the process that change from one week to the next should be investigated as potential sources of variation that could be reduced. Generalizing statistical process control to include multiple sources of variation can greatly extend the type of applications in which control charts are informative and can be used for process improvement. Explicitly modeling and estimating the different variance components helps to identify where in the process the greatest emphasis should be placed for the purpose of variance reduction. Any such reduction in variation leads to improved quality and so understanding the causes of variation and monitoring the process with respect to variation become the foundations for product improvement.

Many examples of SPC for multiple sources of variation appear in the literature. Hahn and Cockrum (1987) consider an example in which batches correspond to production runs of a plastic material. Some run-to-run variation was expected because of significant time delays between production runs with resulting variability in raw materials and ambient conditions. Wheeler and Chambers (1986) give a numerical example involving tensile strength for several heats of steel. Russel et. al. (1974) give examples in clinical laboratory quality control, and Woodall and Thomas (1992) give an example with three components of variation involving integrated circuit processing.

In this paper, the example presented was from the manufacturing of neutron tubes. The multiple sources of variation were week-to-week differences, run-to-run differences within a week, and unit-to-unit differences within a run. However, the results are applicable to many other cases such as those listed above in which there are multiple sources of variation having a hierarchical structure.
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Western Electric Company (1956), Statistical Quality Control Handbook, available from AT&T Technologies, Commercial Sales Clerk, Select Code 700-444, P.O. Box 19901, Indianapolis, IN 46219.


Table 1. Control Chart Constants for Standard Deviation Charts (Subgroups of 2 to 10)

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Op. 6 Field 89 - XDIM Modelled

Within Run Standard Deviation
Exhaust System = 4

Figure 1
Figure 2

Op. 6 Field 89 - XDIM Modelled

Run to Run Standard Deviation
Exhaust System = 4
Op. 6 Field 89 - XDIM Modelled

Averages - Weekly and by Run
Exhaust System = 4

Figure 3