Four Aerodynamic Prediction Schemes for Vertical-Axis Wind Turbines: A Compendium

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FOUR AERODYNAMIC PREDICTION SCHEMES FOR VERTICAL-AXIS WIND TURBINES: A COMPENDIUM

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ABSTRACT

Four aerodynamic models/design tools used to predict the performance for the vertical-axis wind turbine (VAWT) are described. These models are all based upon the conservation of momentum, and are either currently being used at Sandia Laboratories or have been recently used there. A number of comparisons both with the experiments and between the mathematical treatments is made.
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Nomenclature

\( A_s \)  
Turbine swept area

\( c \)  
Blade chord

\( C_P \)  
Power coefficient, \( Q_\omega / (1/2) \rho \omega V_\infty^3 A_s \)

\( \beta \)  
Power coefficient, \( Q_\omega / (1/2) \rho \omega A_s (R_\omega)^3 = C_P / X^3 \)

\( L \)  
Blade length

\( N \)  
Number of blades

\( Q \)  
Turbine torque

\( R \)  
Turbine maximum radius

\( \text{Re}_c \)  
Chord Reynolds number, \( \rho \omega c / \mu_\infty \)

\( V_\infty \)  
Freestream velocity

\( X \)  
Turbine tip-speed ratio, \( \omega V_\infty / R_\omega \)

\( \mu_\infty \)  
Freestream viscosity

\( \rho_\infty \)  
Freestream density

\( \omega \)  
Turbine rotational speed

\( \sigma \)  
Solidity, \( NcL / A_s \)
Summary

Schemes for predicting aerodynamic performance of vertical-axis wind turbines in use at Sandia Laboratories are described. The power coefficients generated by these models are compared to those measured in a wind tunnel using a 2-metre turbine and free-air data of a 5-metre and a 17-metre turbine. All the prediction schemes are based upon the conservation of momentum and vary in complexity. The simplest aerodynamic model, SIMOSS, yields the poorest agreement with experimental data. Schemes involving more complexity better represent the data. The choice of method will depend upon the intended application.

Introduction

A large collection of models exist which predict aerodynamic performance of vertical-axis wind turbines. These models range from the relative simplicity of calculations based on conservation-of-momentum principles to those using rather complex vortex representations and may or may not include unsteady effects. Reference 1 lists and briefly describes many of the quasisteady approaches while Reference 2 discusses some of the dynamic treatments. Sandia Laboratories has used four basic computer models to predict overall aerodynamic performance of vertical-axis wind turbines; each has its physical basis in the conservation-of-momentum principle and is quasisteady. The assumptions and descriptive equations are quite reminiscent of the classical actuator-disc model of propeller aerodynamics. The four models are known as SIMOSS, DART, DARTER and PAREP.

Model Description

SIMOSS (Simple Momentum Single Streamtube)

The simplest momentum model takes the rotor to be enclosed in a single streamtube. Wind velocity across the area swept by the rotor is assumed constant. By choosing some value of this velocity, a combination of the simple actuator-disc momentum model and blade-element theory will give the far-field wind speed, turbine power, torque, and drag for a turbine with given blade characteristics, rotational speed, and geometry. In SIMOSS, an iteration is incorporated which allows the far-field wind speed to be treated as an input while wind velocity across the rotor-swept
area is calculated. The blade planforms that SIMOSS will handle are parabolic, straight blade, and trapezoidal.

**DART (Darrieus Turbine)**

DART differs from SIMOSS in that a multiple streamtube system is used; i.e., the swept area is modeled by an arbitrary number of adjacent and aerodynamically independent areas over which the conservation-of-momentum principle is applied. Advantages gained by this refinement stem from allowing disc-area-wind speeds to vary from section to section. Specifically, this enables airfoil-section data based on elemental Reynolds numbers and wind-shear effects to be included. The computational approach is somewhat different from that of the single streamtube version. For a given geometry, rotational speed, and ambient-wind speed, it is necessary to iterate between the conservation-of-momentum expression and the blade-element formulae to obtain the streamtube wind speed at the rotor. This must be done for each individual streamtube. Rotor power, torque, and drag are obtained by summing the contributions of each tube. DART is restricted to sinusoidal blade planforms.

**DARTER (DART, Elemental Reynolds Number)**

The differences between DART and DARTER are that in the latter there is a capability of using airfoil data based on elemental Reynolds numbers (as opposed to using data based on a single, a priori-determined Reynolds number in DART) and of examining a number of different blade planforms. DARTER may treat (i) troposkein, (ii) straight-line/circular-arc, (iii) parabolic, or (iv) straight-blade geometries.

**PAREP (Parametric Representation)**

The last model, PAREP, is more of a design tool than a mathematical model. The multiple-streamtube models do a reasonable job of predicting maximum power coefficients ($C_p$'s and $K_p$'s) and the tip-speed/ambient-wind-speed ratios at which they occur. However, for high blade loadings and high solidities, there is often considerable discrepancy between theory and experiment away from the maximum power coefficients. PAREP is a sequence which combines theory with results of wind-tunnel testing to better represent overall aerodynamic performance. Specifically, PAREP operates by first referencing curve fits of relevant output from DARTER for a given turbine design. This is due to the fact that, over a reasonable range of blade Reynolds number and turbine solidity, $C_{P_{\text{max}}}$, $X @ C_{P_{\text{max}}}$, $K_{P_{\text{max}}}$, and $X @ K_{P_{\text{max}}}$ can be expressed as simple functions of $Re$ and $\sigma$.

Next, 2-metre wind-tunnel-test results are used to provide the value of $X$ at "runaway" (high-speed ratio at which zero aerodynamic torque is produced). After the user chooses some low value of the zero aerodynamic torque-speed ratio (between one and two, as the final results are relatively insensitive to a choice in this range), a curve is fit between the two zero-power speed-ratio points which passes through the two power coefficient maxima. The shape of this curve is predicted from the 2-metre wind-tunnel-test results.

*Formerly called CPPARM/TVSV*
Figures 1 through 5 summarize the power coefficient predicting capability of three computer models. The DART model is not compared directly since it is no longer operational at Sandia Laboratories; it was replaced by the improved version known as DARTER. Reference 5 presents a comparison of the DART model with some early wind-tunnel data. The standards of comparison are actual data gathered during the testing of Sandia's 2-, 5- and 17-metre vertical-axis wind turbines with near unit height-to-diameter ratios (H/D ≈ 1). These turbines cover a broad range of solidities (0.13 to 0.30), rotational speeds (29.6 to 600 rpm), number of blades (2 or 3), and turbine heights (2, 5, and 17 m). In addition, both wind-tunnel and field-test operations are included.

Figures 1 and 2 present comparisons of the computer models with wind-tunnel data for the 2-metre vertical-axis wind turbine. The two figures differ in turbine solidity and rotational speed. Two sets of aerodynamic section data are used. The primary set was taken in the Wichita State University (WSU) wind tunnel for an NACA-0012 airfoil. The section data for Reynolds numbers of $3.5 \times 10^5$ and $5.0 \times 10^5$ are listed in Reference 9. The secondary set was recently made available by Sharpe (Reference 12) for the same section. The latter set differs from the WSU data in that it extends to a much lower Reynolds number ($4.0 \times 10^4$ vs $3.6 \times 10^5$) and much of it was gathered in a high-turbulence tunnel. DARTER predictions using Sharpe data show better agreement with experiments than do the DARTER predictions using WSU measurements, indicating the importance of using drag information for the proper Reynolds numbers. The results show the SIMOSS model to overpredict the measured maximum-power coefficients and to grossly overpredict the values of the runaway tip-speed ratio (highest ratio at which zero aerodynamic torque is produced). These calculations were based upon an equatorial blade Re and could be improved by using some average Re. DARTER more closely predicts the maximum-power coefficients but overpredicts the runaway tip-speed ratio in all but the low solidity Sharpe-based calculation. PAREP produces the best overall representation of the actual data.

The comparisons of the models with free-air data obtained for the 5-metre vertical-axis wind turbine are presented in Figure 3. The results of SIMOSS and DARTER are similar to the results for the higher 2-metre turbine. Two curves are presented for PAREP because of the geometry of the 5-metre turbine blades. They were not of continuous cross section from hub-to-hub, but rather each blade was made in three sections with the two straight sections near the rotating axis not of aerodynamic cross section. Curve 1 is the PAREP model with the inclusion of the straight sections which had higher aerodynamic drag than the airfoil cross section. Curve 2 is the PAREP model assuming hub-to-hub airfoil cross-section blades. PAREP, as expected, gives the best representation of the actual field-test data.
Figure 1. A Comparison of the Aerodynamic Prediction Schemes with Wind Tunnel Data of a 2-metre Turbine at a Reynolds Number of $2.0 \times 10^5$ and a Solidity of 0.13

Figure 2. A Comparison of the Aerodynamic Prediction Schemes with Wind Tunnel Data of a 2-metre Turbine at a Reynolds Number of $2.8 \times 10^5$ and a Solidity of 0.25
Figure 3. A Comparison of the Aerodynamic Prediction Schemes with Free-Air Data of a 5-metre Turbine at a Reynolds Number of 4.0 x 10^5 and a Solidity of 0.26

The 17-metre vertical-axis wind-turbine data obtained in free-air testing are presented in Figures 4 and 5 for two different rotational speeds and are compared with the models. The results are similar to the previous low-solidity findings with SIMOSS overpredicting the maxima and the runaway tip-speed ratio. DARTER more closely predicts the maximum-power coefficients and properly predicts the runaway tip-speed ratios only when using the Sharpe compilation of airfoil data.

Conclusion

Generally speaking, SIMOSS overestimates both the maximum-power coefficients and the runaway tip-speed ratios, with the latter's disagreement increasing with increasing solidity. This is also a problem with DARTER, but power-coefficient maxima are more closely computed. These uniformly overestimated maxima are the same as those of PAREP, as expected. The improved runaway-ratio predictions seem reasonably good for the free-air running 5- and 17-metre turbines, even though these speeds are based upon wind-tunnel operation of the 2-metre models. For low solidities, runaway ratios are well predicted by Sharpe-based DARTER computations. The tip-speed ratios at which positive aerodynamic torque is first produced are generally between one and two and are so predicted by all computational schemes. The recently acquired NACA-0012 airfoil data of Sharpe used with DARTER best predicts the wind-tunnel and free-air experimental wind-turbine data for low solidities.
Figure 4. A Comparison of the Aerodynamic Prediction Schemes with Free-Air Data of a 17-metre Turbine at a Reynolds Number of $1.08 \times 10^6$ and a Solidity of 0.14

Figure 5. A Comparison of the Aerodynamic Prediction Schemes with Free-Air Data of a 17-metre Turbine at a Reynolds Number of $1.35 \times 10^6$ and a Solidity of 0.14
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